Online shopping and platform design with ex ante registration requirements

Online Appendix

June 17, 2016

This supplementary appendix to the article "Online shopping and platform design with ex ante registration requirements" investigates the firms' equilibrium registration requirements in the presence of competition between sellers. Section B extends the baseline model of Section 3 of the article. Section C offers some brief observations on discounts, dynamics, and the value of consumer information, in a model with competing sellers. Section D extends the model with price commitment (see Remark 1 in the main article) to the case of competing sellers. Section E contains the proofs of the main propositions of this appendix.

B The baseline model with competition

In the baseline model of Section 3, a monopoly firm benefits from requiring ex ante registration, but consumers prefer the firm to offer the option of guest checkout. Therefore, competitors that do not require ex ante registration may attract (some of) the consumers, which may change the profitability of ex ante registration requirements. This appendix shows, however, that the logic of ex ante registration requirements prevails even when firms face competition.

Suppose that there are $N \geq 2$ firms that all produce an identical product at marginal cost of zero. The mass of consumers consists of a share of 'loyal' consumers who only consider to buy at a particular firm and a remaining share of non-committed consumers. We assume that a share β of the consumers is loyal to firm $i = 1, ..., N$ where $0 \leq \beta \leq 1/N$; the share of non-committed consumers is denoted by $\beta_0 := 1 - N\beta$. Brand loyalty can be a consequence of complementarities with other products, for instance, or of switching costs, network effects, consumption choices of friends, or simple unawareness of the presence of competitors.[1] Each consumer has unit demand and a valuation θ which is distributed according to F, both for loyal and for non-committed consumers.

The three-stage game of Section 3 only needs slight modifications. In stage 1, the firms simultaneously and independently make their platform choice $r_i \in \{ExA, G\}$.[2] In stage 2, the platform choices become common knowledge and firm $i \in \{1, ..., N\}$ chooses a price $p_i \geq 0$. For stage 3, we have to distinguish between loyal and non-committed consumers. Loyal consumers decide whether to trade with one particular firm only, just as in the monopoly case. If this firm does not require ex ante registration, loyal consumers observe the price and their valuation θ and decide whether to buy. If this firm requires ex ante registration, loyal consumers decide whether to register, in which case they learn θ and the price and may buy.[3] Non-committed consumers observe the prices of the firms that do not require ex ante registration. Moreover, if there is a firm with $r_i = G$, non-committed consumers learn their valuation θ. Non-committed consumers decide whether to register at a firm (if required) and decide whether and where to buy.[4] As before, consumers who register incur a cost k_R; consumers who buy using the guest checkout (if offered) incur a cost k_G.

The equilibrium analysis builds on the following observations. First, if a firm i requires ex ante registration ($r_i = ExA$), the optimal price choice in stage 2 is $p(0)$, where $p(k)$ is given by

$$p(k) = \frac{1 - F(p(k) + k)}{F'(p(k) + k)}, \tag{26}$$

just as in equation (1) of Section 3.[5] Anticipating this price, consumers would register at

[1] For early papers on price competition with brand loyalty see Rosenthal (1980) and Narasimhan (1988). A similar structure emerges when a share of consumers is uninformed about the existence of other firms (Varian 1980, Baye et al.1992). Brand loyalty can be explained by switching cost, more specifically, for instance, by costly learning how to use new products, complementarities to other purchased products and network effects; for an overview of reasons for brand loyalty see Klemperer (1995). In Baye and Morgan (2001) loyalty emerges from local segregation of markets and can be broken down by creating a virtual marketplace on the internet. See also Baye and Morgan (2009) for a model of price competition when consumer loyalty is endogenous and affected by advertising.

[2] For brevity, we ignore the option $r = ExP$ throughout Sections B and D of this appendix. As in the baseline model, allowing firms to choose $r = ExP$ does not yield any interesting additional insights, unless one considers repeat purchases or an informational value of registration (see Section C).

[3] Thus, if $\beta \to 1/N$, the firms' decisions become completely independent and the equilibrium platform choices are exactly as characterized in Proposition 1 of Section 3.

[4] In case two or more firms require ex ante registration, the non-committed consumers can register at one firm first and observe this firm's price but are allowed to register at another firm afterwards and finally decide where to buy.

[5] There are again equilibria in which consumers believe that the firm chooses a very high price such that it never pays off to register at this firm and therefore, this firm has indeed no incentive to deviate from a very high price. As in the monopoly case, equilibrium refinements can eliminate these equilibria.

this firm only if $k_R \leq \bar{u}(0)$, where

$$\bar{u}(0) = \int_{p(0)}^{\infty} (\theta - p(0))\, dF(\theta) \tag{27}$$

as in equation (5) of Section 3; moreover, non-committed consumers expect the same surplus $\bar{u}(0)$ at either firm that requires ex ante registration. Second, if two or more firms offer the option of guest checkout, there is price competition for the non-committed consumers; this yields equilibrium prices below $p(0)$ such that non-committed consumers are strictly better off when trading with a firm i that chooses $r_i = G$ than when registering at a firm j with $r_j = ExA$. These observations lead to the following main result of this section.

Proposition 7 *(i) Suppose that $k_R \leq \bar{u}(0)$, where $\bar{u}(0)$ is defined in (27). If*

$$\beta \geq \frac{1}{N-1}\left(1 - \frac{\pi(0)}{N\pi(k_G)}\right), \tag{28}$$

in equilibrium all firms require ex ante registration. If

$$0 < \beta < \frac{1}{N-1}\left(1 - \frac{\pi(0)}{N\pi(k_G)}\right), \tag{29}$$

there are N equilibria such that exactly one firm offers the option of guest account and all other firms require ex ante registration.[6]
(ii) If $k_R > \bar{u}(0)$, all firms $i = 1, ..., N$ choose $r_i = G$.

Proof. See Section E.1 below. ■

Price competition between firms lowers their expected profits if two or more firms do not require ex ante registration.[7] All profits from selling to the non-committed consumers are competed away, and firm i that chooses $r_i = G$ ends up with an expected equilibrium payoff equal to $\beta\pi(k_G)$ (which is what it can guarantee itself when selling to its loyal consumers

[6] Under the condition (29), there are also equilibria where (some) firms randomize their platform choices in stage 1, including a symmetric equilibrium in which all N firms randomize their registration requirements, choosing $r_i = ExA$ with some probability α and $r_i = G$ with probability $1 - \alpha$ (see the proof in Appendix E.1).

[7] In the proof of Proposition 7 we show that in this case, the equilibrium pricing decisions are in mixed strategies whenever $\beta > 0$. Intuitively, prices do not drop down to marginal costs since firms can make positive profits by selling to their loyal consumers only. Nevertheless, firms would like to marginally undercut their competitors in order to gain all non-committed consumers. The equilibrium of Bertrand price competition with a share of loyal consumers has been derived and applied by Narasimhan (1988) for the case of two firms, and similar structures have been analyzed, for instance, in the context of price competition with informed and uninformed consumers (Varian 1980; Baye et al. 1992).

only), where

$$\pi(k) = (1 - F(p(k) + k))\, p(k) \tag{30}$$

as in equation (2) of Section 3. But if firm i requires ex ante registration, it gets a profit $\beta\pi(0) > \beta\pi(k_G)$: Even though only loyal consumers consider buying at firm i in this case, the same argument as in the baseline model (Proposition 1) shows that firm i is strictly better off when those consumers register ex ante.

If the firms' share of loyal consumers is sufficiently high then all firms choose an ex ante registration requirement in equilibrium and set prices equal to $p(0)$. The non-committed consumers register at one randomly selected firm and only consider buying at this firm since they correctly anticipate that the prices at the other firms will not be lower.[8] If a firm i deviates to 'no ex ante registration' and chooses $r_i = G$, it optimally sets a price $p(k_G) < p(0)$ and gets all non-committed consumers; but it loses the advantage of ex ante registration of its loyal consumers. Hence, a deviation to $r_i = G$ is profitable only if β becomes small such that the gain from additional non-committed consumers outweighs the lower profit extracted from the loyal consumers. Such a deviation is, however, profitable for at most one firm: Price competition about non-committed consumers deters all remaining firms $j \neq i$ from choosing $r_j = G$ whenever $\beta > 0$ (however small). In other words, in any equilibrium where firms do not randomize their choices of registration requirements, all firms except possibly one require ex ante registration.

As long as $\beta > 0$, Proposition 7 characterizes the complete set of equilibria involving registration policy choices in pure strategies, even when β is infinitesimally close to zero. The type of equilibrium in which $N - 1$ firms require ex ante registration continues to exist for $\beta = 0$, but in this case there are (multiple) additional equilibria since a firm i makes zero profits both in case of $r_i = ExA$ and when deviating to $r_i = G$ (provided that at least one other firm j chooses $r_j = G$, as none of the consumers would register at a firm if there is another firm with $r_j = G$). Therefore, the case of $\beta = 0$ is a special case in which there are multiple equilibria characterized by $m \in \{1, ..., N\}$ firms choosing $r_j = G$ and the remaining $N - m$ firms choosing $r_i = ExA$.

A general message of the case of price competition is that firms with a high share of

[8] Proposition 7 is similar to the Diamond (1971) paradox that arbitrarily small search costs imply equilibrium prices that differ drastically from marginal costs. Similarly, in our setting, arbitrarily small frictions in the form of registration costs lead to equilibrium prices above marginal costs. If (29) holds, however, prices paid by the uncommitted consumers are smaller than the monopoly price (but still strictly higher than marginal costs). Moreover, our result is different from Diamond (1971) since it relies on both registration costs ($k_R > 0$) and committed consumers ($\beta > 0$). Intuitively, while firms may not want to deviate in prices given that all firms choose $r = ExA$, the registration requirements add another dimension in which firms may deviate, which would have its analogy in Diamond's model (though being substantially different) when allowing firms to reduce the search costs to zero and in this way restore price competition in equilibrium.

loyal consumers choose ex ante registration requirements while firms with no (of few) loyal consumers do not require ex ante registration, which can be most easily seen in the following example.

Remark 4 *Let $1 \leq n \leq N-2$ and suppose that firms $i \in \{1,...,n\}$ each have a share $\beta \in (0, 1/n]$ of loyal consumers but firms $j \in \{n+1,...,N\}$ do not have any loyal consumers. If $k_R \leq \bar{u}(0)$, there is an equilibrium in which firms $i = 1,...,n$ choose $r_i = ExA$ and all other firms $j = n+1,...,N$ choose $r_j = G$.*

If two or more firms j do not have any loyal consumers, they offer the option of guest checkout, choose prices equal to marginal costs and, hence, realize zero profits, but they cannot do better by deviating to $r_j = ExA$ since in this case none of the non-committed consumers would register at their shop. Firms i with loyal consumers are, thus, strictly better off when choosing $r_i = ExA$ (in which case they get a profit of $\beta\pi(0) > 0$).

C Remarks on discounts, dynamics, and informational value of consumer registration

Section 6 of the main article considered the incentive of a monopoly firm to use discounts as a means to increase the consumers' willingness to register. Similar effects can be derived vis-à-vis the loyal consumers in the case of competition. Moreover, discounts may also be used to attract the non-committed consumers.[9] As a simple illustration, consider the case of Proposition 7(i) and suppose that firms $j \neq i$ do not use discounts. Then, firm i can attract additional non-committed consumers by offering them an arbitrarily small discount. This will increase firm i's profit even when it cannot target the discount specifically to the non-committed consumers. As in Section 6, the firm cannot offer discounts to all consumers since this would lead to a price increase by the same amount. But since under the assumptions of Proposition 7(i) loyal consumers register even without discounts, the registered consumers will consist of consumers with and without discounts; hence, non-committed consumers who are offered a discount anticipate that their effective price at firm i is reduced, and prefer to register at firm i.

In a multi-period model with non-monetary costs of registration, some degree of consumer loyalty can also emerge endogenously. Intuitively, whenever a consumer already has an account at a firm, he is willing to pay a higher price at that firm if, at other firms, he needs to

[9] See also Shaffer and Zhang (2002) on price competition with loyal consumers when, after setting their prices, firms can target promotions to certain customers and induce consumers to switch to their brand.

register (or use guest checkout) when buying. This generates a lock-in effect which mitigates the price competition and is, hence, profitable for the firms. In fact, when extending the two-period model of Section 4 of the main article to the case of N firms (without exogenously loyal consumers, that is, $\beta = 0$), there can be an equilibrium in which all N firms do not offer the option of guest checkout in period t but require registration ex post, that is, whenever a consumer decides to buy. Although deviating could attract additional consumers in the current period, requiring consumers to register leads to higher profits in future periods.

If firms derive an informational value from consumer registration (as in Section 5 of the main article), this strengthens their incentive to require registration also in the presence of competition. In Proposition 7(i), the range of β for which all firms require ex ante registration in equilibrium would be enlarged if firms value the information that consumers provide when registering. Overall, the incentive to use ex ante registration requirements caused by the sunk cost advantage of registration carries over to the case of competition and its value is reinforced when incorporating dynamic aspects and repeat purchases or an informational value of consumer registration.

D Price commitment and ex ante registration under competition

Above we assumed that, when a firm requires registration, unregistered consumers cannot observe its price. Here, we consider the case where prices are binding commitments for the firms and observable to all consumers, as in Remark 1 concerning the monopoly case.

The analysis of competition in Section B yielded two main insights. First, even under price competition firms may be able to sustain prices equal to the monopoly price $p(0)$ (as in the equilibrium in which all firms require ex ante registration). Second, ex ante registration requirements continue to be profitable in the model with $N \geq 2$ firms. In this section, we show that the result of monopoly prices in equilibrium hinges on the assumption that, with ex ante registration requirements, prices are not observable for the consumers. Just as informative advertising can offer a solution to the Diamond (1971) paradox and related price obfuscation effects, observability of (and commitment to) prices in case of ex ante registration requirements fosters price competition and leads to lower profits. Ex ante registration requirements, however, are still prevalent in equilibrium; their profitability does not depend on the price obfuscation that prices of firms j with $r_j = ExA$ are unobservable to non-registered consumers.

Formally, we consider the model with $N \geq 2$ firms described in Section B above, except that we modify the timing of the game by assuming that after the firms have chosen their prices, all prices (including those of the firms that require ex ante registration) are observed by the consumers (as in Remark 1 on the monopoly case with price commitment).[10] Let

$$p_C(k) = \arg\max_p (1 - F(p))\, p \quad \text{s.t.} \quad \int_p^\infty (\theta - p)\, dF(\theta) \geq k$$

denote the optimal price of a monopoly firm that chooses ex ante registration and commits to a price under the constraint that consumers with registration costs k are willing to register at this price. If k_R is low ($k_R \leq \bar{u}(0)$) then the constraint on consumer surplus does not bind such that $p_C(k_R) = p(0)$. If k_R is high ($k_R > \bar{u}(0)$) then the firm has to lower its price in order to make the consumers willing to register. The following proposition addresses the case in which the monopoly price under $r = ExA$ is higher than the monopoly price under $r = G$ (the latter is equal to $p(k_G)$ as given by (26)). Formally, this is the case when $k_R < \bar{k}$, where $\bar{k} \in (\bar{u}(0), E(\theta))$ is defined such that

$$p_C\left(\bar{k}\right) = p(k_G).$$

Proposition 8 *Consider the case in which the firms commit ex ante to their prices and suppose that $k_R < \bar{k}$.*
(i) In any equilibrium where firms do not randomize their registration requirements, all firms except possibly one will require ex ante registration.
(ii) There exists a threshold $\bar{\beta} \in (0, 1/N)$ such that in equilibrium all firms require ex ante registration if $\beta > \bar{\beta}$.

Proof. See Section E.2 below. ■

Proposition 8 shows that as in the case without price commitment, all firms except possibly one will require ex ante registration in any equilibrium with platform choices in pure strategies. If all firms require ex ante registration and the firms' prices are observable to all consumers, this will yield price-setting in mixed strategies and prices below the monopoly price $p_C(k_R)$. Nevertheless, the firms' realized profits are sufficiently high to make a deviation to $r_j = G$ unattractive. In particular, as soon as one firm chooses $r_i = G$, no other firm $j \neq i$ would like to deviate from $r_j = ExA$ and lose the advantage from ex ante registration of their loyal consumers. If β is sufficiently high, all firms require ex ante registration in equilibrium, but for lower β there can be an equilibrium in which exactly one firm i chooses $r_i = G$ and all other firms $j \neq i$ choose $r_j = ExA$ (for more details see the proof of Proposition 8).

[10] As in Section B above, we ignore the option $r_i = ExP$ for brevity, as it does not provide interesting additional insights.

Proposition 8 confirms the advantage from ex ante registration requirements when price obfuscation effects are absent. Compared to Proposition 7, the range of β in which all firms require ex ante registration becomes smaller but the range of k_R for which ex ante registration requirements are feasible is enlarged (which follows from $\bar{k} > \bar{u}(0)$), in case firms commit to their prices before consumers decide whether to register.[11] Thus, while the absence of price obfuscation effects leads to lower prices, price commitment also makes it easier for firms to induce consumers to register ex ante.

E Additional proofs

E.1 Proof of Proposition 7

First we derive the equilibrium prices and profits for given platform choices $(r_1, ..., r_N)$. Suppose that all firms choose $r_i = ExA$. Then, given that a positive share of consumers register, it is optimal for each firm to choose a price $p(0)$. Anticipating the firms' pricing decisions, loyal consumers register if and only if $k_R \leq \bar{u}(0)$. Non-committed consumers are indifferent between registration at either firm and randomly select one of the firms. Thus, if $k_R \leq \bar{u}(0)$, firm i gets its loyal consumers and a share $1/N$ of the non-committed consumers and, hence, a profit of $(\beta + \beta_0/N)\,\pi(0) = \pi(0)/N$. If $k_R > \bar{u}(0)$, no consumer registers and the firms' profits are zero.

Now suppose that exactly one firm chooses $r_i = G$ and all other firms $j \neq i$ choose $r_j = ExA$. For firm i with $r_i = G$, it is optimal to set $p_i = p(k_G)$, which, by Assumption 1 (the distribution F of valuations has a monotone hazard rate), is strictly smaller than $p(0)$. Observing p_i and anticipating the prices of firms $j \neq i$, all non-committed consumers consider only buying at firm i. Therefore, firm i has no incentive to deviate to another price ($p(k_G)$ is the optimal price even in the absence of competition) and gets a profit of $(\beta + \beta_0)\,\pi(k_G)$ from selling to its loyal consumers and to the non-committed consumers. All other firms j (with $r_j = ExA$) can only sell to their loyal consumers; they realize a profit of $\beta\pi(0)$ if $k_R \leq \bar{u}(0)$ and a profit of zero otherwise.

Finally, suppose that $m \geq 2$ firms choose $r_i = G$ and the remaining firms choose $r_j = ExA$. The resulting subgame equilibrium is summarized in the following lemma.[12]

[11] There are parameter values (β, k_R) for which $r_i = ExA$ for $i = 1, ..., N$ occurs in equilibrium in case of price commitment, but does not constitute an equilibrium with unobservable prices, and vice versa.

[12] Baye et al. (1992) show that the game of price competition with uninformed (loyal) and informed (non-committed) consumers has a unique symmetric equilibrium as well as a continuum of asymmetric equilibria in mixed strategies. Since the equilibria are payoff-equivalent, the following lemma focuses on the equilibrium in which the firms i with $r_i = G$ use symmetric mixed strategies, without affecting the consequences for the equilibrium platform choices.

Lemma 2 *Let $2 \leq m \leq N$ and suppose that firms $i = 1, ..., m$ choose $r_i = G$ and the remaining firms choose $r_j = ExA$. In the symmetric equilibrium, firm $i \in \{1, ..., m\}$ randomizes according to*

$$B(p_i) = \begin{cases} 0 & \text{if } p_i \leq \underline{p} \\ 1 - \left(1 - \frac{\beta+\beta_0}{\beta_0} + \frac{\beta\pi(k_G)}{\beta_0(1-F(p_i+k_G))p_i}\right)^{\frac{1}{m-1}} & \text{if } \underline{p} < p_i < p(k_G) \\ 1 & \text{if } p_i \geq p(k_G) \end{cases} \tag{31}$$

where $\underline{p}$ is defined as the solution to

$$(1 - F(\underline{p} + k_G))\underline{p} = \frac{\beta}{\beta + \beta_0}\pi(k_G) \tag{32}$$

that fulfills $\underline{p} < p(k_G)$. Firm $j \in \{m+1, ..., N\}$ chooses $p_j = p(0)$. The expected equilibrium profit of firm $i \in \{1, ..., m\}$ is equal to $\beta\pi(k_G)$. The expected equilibrium profit of firm $j \in \{m+1, ..., N\}$ is equal to $\beta\pi(0)$ if $k_R \leq \bar{u}(0)$ and equal to zero otherwise.

Proof. Consider first firms $j \in \{m+1, ..., N\}$ with $r_j = ExA$. For those firms, equilibrium prices and profits follow as in Section 3. If a positive mass of consumers register then j's optimal price is $p(0)$. If $k_R \leq \bar{u}(0)$, all loyal consumers register at firm j, which yields a profit of $\beta\pi(0)$; otherwise, no consumer registers and j gets zero profit. Due to $p(0) > p(k_G) \geq p_i$ for $i \in \{1, ..., m\}$, non-committed consumers never consider registering/buying at firms $j \in \{m+1, ..., N\}$.

Firms $i \in \{1, ..., m\}$ randomize according to $B(p_i)$ on the support $[\underline{p}, p(k_G)]$. Using (32), it is straightforward to verify that $B(\underline{p}) = 0$; moreover, $B(p_i)$ is strictly increasing on $(\underline{p}, p(k_G))$ with $B(p(k_G)) = 1$. Due to the regularity assumptions on F, $\underline{p}$ is uniquely defined by (32); it approaches zero for $\beta \to 0$ and approaches $p(k_G)$ for $\beta \to 1/N$.

Consider firm $i \in \{1, ..., m\}$ and suppose that all other firms $l \in \{1, ..., m\}$, $l \neq i$ randomize according to $B(p)$ in (31). If i chooses a price $p_i \in [\underline{p}, p(k_G)]$, it sells to its loyal consumers at this price; in addition, it sells to all non-committed consumers if and only if p_i is lower than the prices of all other firms, that is, with probability $(1 - B(p_i))^{m-1}$. (Recall that firms $j \in \{m+1, ..., N\}$ choose a price $p(0) > p(k_G) \geq p_i$ and will never sell to the non-committed consumers.) Thus, firm i's expected profit when choosing $p_i \in [\underline{p}, p(k_G)]$ is equal to

$$\begin{aligned}
& \left(\beta + (1 - B(p_i))^{m-1} \beta_0\right)(1 - F(p_i + k_G))\, p_i \\
= \; & \left(\beta + \left(1 - \frac{\beta + \beta_0}{\beta_0} + \frac{\beta \pi(k_G)}{\beta_0 (1 - F(p_i + k_G))\, p_i}\right) \beta_0\right)(1 - F(p_i + k_G))\, p_i \\
= \; & \beta \pi(k_G)
\end{aligned}$$

and is, hence, independent of p_i. In particular, if $p_i = \underline{p}$, i sells to a share $\beta + \beta_0$ of the consumers; with (32) this yields a profit of $\beta\pi(k_G)$. If $p_i = p(k_G)$, i only sells to its loyal consumers which, again, yields a profit of $\beta\pi(k_G)$. Moreover, prices below $\underline{p}$ and above $p(k_G)$ lead to a strictly lower profit. Since i is indifferent between all $p_i \in [\underline{p}, p(k_G)]$, randomization according to $B(p_i)$ is a best reply. ■

If two or more firms choose $r_i = G$, these firms' equilibrium prices cannot be in pure strategies. To see why, suppose that $m = 2$ and that $p_1 = \tilde{p} > 0$. Firm 2's best reply is $p_2 = p(k_G)$ if $\tilde{p} > p(k_G)$ and $p_2 = \tilde{p} - \varepsilon$ otherwise, $\varepsilon > 0$ infinitesimally small.[13] But then, firm 1 strictly prefers $p_1 = p_2 - \delta$, $\delta > 0$ infinitesimally small, over $p_1 = \tilde{p}$. Moreover, p_1 cannot be zero in a pure strategy equilibrium. If $p_1 = 0$, firm 1 has zero profits, but it can achieve at least $\beta\pi(k_G) > 0$ by setting $p_1 = p(k_G)$. Thus, the equilibrium must be in mixed strategies. In any equilibrium, firms will not choose prices higher than $p(k_G)$, which is the price a firm would choose in the absence of competition, or if the other firms' prices are higher. Using standard techniques in auction theory, it can be shown that in the unique symmetric equilibrium, the firms randomize continuously on an interval $[\underline{p}, p(k_G)]$ with $\underline{p} < p(k_G)$.[14] Since $p(k_G) < p(0)$, the non-committed consumers never register at a firm j with $r_j = ExA$ since they correctly anticipate the higher price of this firm. The firms' expected profits depending on the number of firms with $r_i = G$ are summarized in Table 1.

We are now in a position to prove Proposition 7. Part (ii) (the case where $k_R > \bar{u}(0)$) is straightforward: The profit of any firm j choosing $r_j = ExA$ is zero, but $r_j = G$ leads to a profit of at least $\beta\pi(k_G) > 0$ so that all firms offer the option of guest checkout.

It remains to prove part (i) (where $k_R \leq \bar{u}(0)$). Suppose that inequality (28) holds, and all firms $j \neq i$ choose $r_j = ExA$. Consider the choice of firm i. Under $r_i = ExA$, i gets an expected profit of $(\beta + \beta_0/N)\,\pi(0) = \pi(0)/N$. If i deviates to $r_i = G$, its expected profit is $(\beta + \beta_0)\,\pi(k_G) = (1 - (N-1)\beta)\,\pi(k_G)$. Thus, i has no incentive to deviate if and only if

$$\pi(0)/N \geq (1 - (N-1)\beta)\,\pi(k_G),$$

[13] To be precise, due to the continuous strategy space, we have to interpret the best reply as an $\varepsilon-$best reply.

[14] Technically, $\underline{p}$ is obtained such that a firm's expected profit when choosing $p = \underline{p}$ is exactly equal to its expected profit when choosing $p = p(k_G)$ and selling only to its loyal consumers.

	Number of firms with $r_i = G$		
	0	1	≥ 2
Profit of a firm with $r_i = G$	-	$(\beta + \beta_0)\,\pi\,(k_G)$	$\beta\pi\,(k_G)$
Profit of a firm with $r_i = ExA$	$(\beta + \beta_0/N)\,\pi\,(0)$	$\beta\pi\,(0)$	$\beta\pi\,(0)$

Note: $\pi(k)$ is defined in equation (30). β is firm i's share of loyal consumers, $\beta_0 = 1 - N\beta$ is the share of non-committed consumers.

Table 1: Summary of the firms' expected profits conditional on the stage 1 platform choices, for the case of $k_R \leq \bar{u}\,(0)$.

which is equivalent to (28). Since under (28), firm i has a dominant strategy to choose $r_i = ExA$ in stage 1, the equilibrium is unique.[15]

Now suppose (29) holds, and suppose that firm 1 chooses $r_1 = G$ and firms $j = 2, ..., N$ choose $r_j = ExA$. As shown above, since (28) is violated, firm 1's profit is strictly higher under $r_1 = G$ than when deviating to $r_1 = ExA$. Moreover, firm $j \in \{2, ..., N\}$ gets an expected profit of $\beta\pi\,(0)$ under $r_j = ExA$ but gets only $\beta\pi\,(k_G) < \beta\pi\,(0)$ when deviating to $r_j = G$ (the proof of the inequality uses the same argument as the proof of Proposition 1). Hence, exactly one firm chooses $r_i = G$ and all other firms $j \neq i$ choose $r_j = ExA$ in the equilibria without randomization of the registration requirements.

Since a firm strictly prefers $r_i = ExA$ over $r_i = G$ as soon as the number of firms with $r_j = G$ is strictly greater than zero, there can be no further equilibrium with platform choices in pure strategies. There are, however, additional equilibria in which firms randomize their choice of registration requirement, in case (29) holds. In particular, there is a symmetric equilibrium in which firms $i = 1, ..., N$ choose $r_i = ExA$ with probability α and $r_i = G$ with probability $1 - \alpha$, where

$$\alpha = \left(\frac{\beta N\,(\pi\,(0) - \pi\,(k_G))}{(1 - \beta N)\,(N\pi\,(k_G) - \pi\,(0))} \right)^{\frac{1}{N-1}}.$$

E.2 Proof of Proposition 8

Let

$$p_C\,(k) = \arg\max_p\,(1 - F\,(p))\,p \quad \text{s.t.} \quad \int_p^\infty (\theta - p)\,dF\,(\theta) \geq k$$

[15] If (28) holds with equality, all firms choose $r_i = ExA$ by our tie-breaking rule.

and

$$\pi_C(k) = \max_p (1 - F(p))\, p \ \text{ s.t. } \int_p^\infty (\theta - p)\, dF(\theta) \geq k.$$

Then, the price $p_C(k_R)$ is equal to $p(0) > p(k_G)$ for $k_R \leq \bar{u}(0)$, and $p_C(k_R)$ is strictly decreasing in k_R for $k_R > \bar{u}(0)$; moreover, $p_C(k_R) \to 0$ if $k_R \to E(\theta)$. Since $p(k_G)$ is independent of k_R, there is a uniquely defined threshold $\bar{k} \in (\bar{u}(0), E(\theta))$ for which $p_C(\bar{k}) = p(k_G)$.

To show part (i), suppose that $k_R < \bar{k}$ and that $m \geq 2$ firms choose $r_i = G$. Since $k_R < \bar{k}$ implies that $p_C(k_R) > p(k_G)$, the equilibrium of the ensuing subgame is similar to the equilibrium in the case without price commitment (see Lemma 2): Firms j with $r_j = ExA$ choose prices equal to $p_C(k_R)$ and firms i with $r_i = G$ randomize their prices just as in (31); for the latter firms, the assumption of price commitment is irrelevant since the prices of firms with $r_i = G$ are observable under both assumptions on observability of prices employed in this appendix. Thus, firms with $r_i = G$ get an expected equilibrium profit of $\beta\pi(k_G)$ and firms with $r_j = ExA$ get an expected equilibrium profit of $\beta\pi_C(k_R)$.

Since $p_C(k_R) > p(k_G)$, $p(k_G)$ is also feasible for firms j with $r_j = ExA$, which implies that

$$\begin{aligned} \beta\pi_C(k_R) &\geq \beta(1 - F(p(k_G)))\, p(k_G) \\ &> \beta(1 - F(p(k_G) + k_G))\, p(k_G) = \beta\pi(k_G). \end{aligned}$$

Hence, a firm i with $r_i = G$ is strictly better off when deviating to $r_i = ExA$, in which it can get at least the profit $\beta\pi_C(k_R)$ from selling to the loyal consumers only. Thus, in any equilibrium in pure strategies on stage 1 at most one firm can choose $r_i \neq ExA$.

To show part (ii), we first characterize the (symmetric) equilibrium on the price-setting stage if all firms choose $r_i = ExA$ and prices are observable. Then, we consider the firms' incentives to deviate from $r_i = ExA$.

Lemma 3 *Suppose that $k_R < E(\theta)$ and that $r_i = ExA$ for all $i = 1, ..., N$. In the symmetric equilibrium, firm i randomizes its price according to*

$$B(p_i) = \begin{cases} 0 & \text{if } p_i \leq \underline{p} \\ 1 - \left(\left(\frac{\pi_C(k_R)}{(1-F(p_i))p_i} - 1\right)\frac{\beta}{\beta_0}\right)^{\frac{1}{N-1}} & \text{if } \underline{p} < p_i \leq p_C(k_R) \\ 1 & \text{if } p_i > p_C(k_R) \end{cases} \tag{33}$$

where $\underline{p}$ is defined as the solution to

$$(\beta + \beta_0)\left(1 - F\left(\underline{p}\right)\right)\underline{p} = \beta\pi_C(k_R) \tag{34}$$

that fulfills $\underline{\text{p}} < p_C(k_R)$*. Firm* i*'s expected equilibrium profit equal to* $\beta\pi_C(k_R)$*.*

Proof. Since $(1 - F(p))\,p$ is zero for $p = 0$, strictly increasing in p for $0 < p < p(0)$ (by Assumption 1) and approaches $\pi_C(k_R)$ for $p \to p_C(k_R)$, there is a unique solution $\underline{\text{p}} < p_C(k_R)$ to (34). Using (33) it is straightforward to verify that $B(\underline{\text{p}}) = 0$ and that $B(p_i)$ is strictly increasing on $(\underline{\text{p}}, p_C(k_R))$ and approaches 1 if $p_i \to p_C(k_R)$. Moreover, at prices below $p_C(k_R)$ all consumers are willing to register ex ante; non-committed consumers will register at the firm that offers the lowest price.

If all firms $j \neq i$ follow (33), firm i's expected profit when choosing a price p_i within the support of $B(p_i)$ is

$$\begin{aligned} & \left(\beta + (1 - B(p_i))^{N-1}\beta_0\right)(1 - F(p_i))\,p_i \\ = \; & \left(\beta + \left(\left(\frac{\pi_C(k_R)}{(1 - F(p_i))\,p_i} - 1\right)\frac{\beta}{\beta_0}\right)\beta_0\right)(1 - F(p_i))\,p_i \\ = \; & \beta\pi_C(k_R), \end{aligned}$$

while firm i gets a strictly lower expected profit when choosing a price $p_i \notin [\underline{\text{p}}, p_C(k_R)]$. Since firm i is indifferent between all $p_i \in [\underline{\text{p}}, p_C(k_R)]$, randomization according to $B(p_i)$ is a best reply.[16] ■

Using the firms' expected profit $\beta\pi_C(k_R)$ in the subgame equilibrium where $r_i = ExA$ for all $i = 1, ..., N$, suppose that firms $j \neq i$ choose $r_j = ExA$ and consider firm i's incentive to deviate to $r_i = G$. The maximum profit that firm i can get under $r_i = G$ is $(\beta + \beta_0)\,\pi(k_G) = (1 - (N - 1)\beta)\,\pi(k_G)$, which i gets if it sells to its loyal consumers plus all non-committed consumers at the optimal price $p(k_G)$. Thus, if

$$\beta \geq \frac{\pi(k_G)}{\pi_C(k_R) + (N - 1)\,\pi(k_G)}, \tag{35}$$

firm i has no incentive to deviate to $r_i = G$. Since $\pi_C(k_R) > \pi(k_G)$, the right-hand side of (35) is strictly between 0 and $1/N$.[17] This shows part (ii).

[16] Similar as in Baye et al. (1992) there are also asymmetric price-setting equilibria in which $m \geq 2$ firms randomize their prices (replacing N by m in (33)) and the remaining firms choose a price $p = p_C(k_R)$ with probability one; these equilibria are payoff-equivalent to the symmetric equilibrium characterized in Lemma 3.

[17] Concerning the case where exactly one firm i chooses $r_i = G$ and all other firms $j \neq i$ choose $r_j = ExA$, our argument above used an upper bound on the profit firm i can attain, but did not prove existence of an equilibrium of the pricing stage. The latter is most easily established for values of β such that firm i gets all non-committed consumers; for this to be the case, β has to be above some threshold $\tilde{\beta} \in (0, 1/N)$, to prevent firms $j \neq i$ with $r_j = ExA$ from deviating to lower prices (details are available on request). This analysis also implies that, when parameters are such that $\beta \geq \tilde{\beta}$ but β is below the threshold defined in (35), there is an equilibrium where exactly one firm i chooses $r_i = G$ and all other firms $j \neq i$ choose $r_j = ExA$.

References

[1] Baye, M. R., Kovenock, D., and de Vries, C. G. (1992). It takes two to tango: Equilibria in a model of sales. Games and Economic Behavior, 4(4), 493-510.

[2] Baye, M. R. and Morgan, J. (2001). Information gatekeepers on the Internet and the competitiveness of homogeneous product markets. American Economic Review, 91(3), 454-474.

[3] Baye, M. R. and Morgan, J. (2009). Brand and price advertising in online markets. Management Science, 55(7), 1139-1151.

[4] Diamond, P. A. (1971). A model of price adjustment. Journal of Economic Theory, 3(2), 156-168.

[5] Klemperer, P. (1995). Competition when consumers have switching costs: An overview with applications to industrial organization, macroeconomics, and international trade. Review of Economic Studies, 62(4), 515-539.

[6] Narasimhan, C. (1988). Competitive promotional strategies. Journal of Business, 61(4), 427-449.

[7] Rosenthal, R. W. (1980). A model in which an increase in the number of sellers leads to a higher price. Econometrica, 48(6), 1575-1579.

[8] Shaffer, G., and Zhang, Z. J. (2002). Competitive one-to-one promotions. Management Science, 48(9), 1143-1160.

[9] Varian, H. R. (1980). A model of sales. American Economic Review, 70(4), 651-659.